AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Module name: Discrete Models of Financial Markets

Academic year: 2019/2020 Code: AMAT-2-103-MF-s ECTS credits: 7

Faculty of: Applied Mathematics

Field of study: Mathematics Specialty: Financial Mathematics

Study level: Second-cycle studies Form and type of study: Full-time studies

Lecture language: Polski i Angielski Profile of education: Academic (A) Semester: 1

Course homepage: —

Responsible teacher: dr hab. Capiński Maciej (mcapinsk@agh.edu.pl)

Module summary

The student will understand mathematical models and the difficulties in making financial mathematics models relevant to real markets.

Description of learning outcomes for module

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Connections with FLO	Method of learning outcomes verification (form of completion)
Social competence: is able to			
M_K001	The student will understand the limitations of mathematical models and the difficulties in making financial mathematics models relevant to real markets.	MAT2A_U08, MAT2A_U15	Activity during classes, Examination
Skills: he can			
M_U001	speaks English at intermediate level (B2) and at a sufficient level to read professional literature	MAT2A_U22	Activity during classes
M_U002	The student will be able to use the CRR model to price basic derivatives (European/American calls and puts).	MAT2A_U18, MAT2A_U16, MAT2A_U10	Activity during classes, Examination
Knowledge: he knows and understands			
M_W001	The student will know how the basic properties of option prices follow from the "no arbitrage formula"	MAT2A_U03, MAT2A_U14, MAT2A_W07, MAT2A_W01, MAT2A_W04	Activity during classes, Examination

M_W002	The student will know how the "no arbitrage" priciple can be used to derive pricing formulae in discrete markets for basic derivatives.	MAT2A_U03, MAT2A_U14, MAT2A_U01	Activity during classes, Examination

Number of hours for each form of classes

	Form of classes										
Suma	Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
60	30	30	0	0	0	0	0	0	0	0	0

FLO matrix in relation to forms of classes

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Form of classes										
		Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
Social competence: is able to												
M_K001	The student will understand the limitations of mathematical models and the difficulties in making financial mathematics models relevant to real markets.	+	+	-	-	-	-	-	-	-	-	-
Skills: he can												
M_U001	speaks English at intermediate level (B2) and at a sufficient level to read professional literature	+	+	-	-	-	-	-	-	-	-	-
M_U002	The student will be able to use the CRR model to price basic derivatives (European/American calls and puts).	+	+	-	-	-	-	-	-	-	-	-
Knowledge: he knows and understands												
M_W001	The student will know how the basic properties of option prices follow from the "no arbitrage formula"	+	+	-	-	-	-	-	-	-	-	-
M_W002	The student will know how the "no arbitrage" priciple can be used to derive pricing formulae in discrete markets for basic derivatives.	+	+	-	-	-	-	-	-	-	-	-

Student workload (ECTS credits balance)

Student activity form	Student workload
Udział w zajęciach dydaktycznych/praktyka	60 h
Preparation for classes	42 h
Realization of independently performed tasks	66 h
Examination or Final test	2 h
Contact hours	5 h
Summary student workload	175 h
Module ECTS credits	7 ECTS

Additional information

Module content

Lectures

Introduction to discrete models

Stocks, bonds/money market account. Short-selling. Portfolios of assets. Long/short positions. Derivatives: forwards, futures and options. Spot and extended markets. Arbitrage transactions: the single step single asset spot market model. Arbitrage in the extended market. The value of a forward contract.

The numeraire approach

Options to exchange assets. Numeraire. The general formula for European calls and puts. Radon-Nikodyn, Bayes and the numeraire change. Option prices and symmetry.

Stopping times and American options

The Snell envelope, stopping times, stopped processes, optimal stopping. Application to American options pricing and hedging.

Complete and incomplete markets.

Complete markets. The trinomial model as a simple example of the incomplete market. A parametrization of risk-neutral measures, superhedging.

Pricing American options in the binomial model

American options: early exercise. Pricing and replicating American options in the binomial model: a naive approach. Bermuda options and other exotic exercises.

Extending CRR

Dividends, dividend yields and storage costs. Forward price formulae. Arbitrage and no-arbitrage assets, market prices and the “no extra cash flows” condition. Pricing options on currencies, stock indices and dividend paying stocks in the CRR model.

General multi-step discrete models

Adapted and predictable processes. Trading strategies. Self-financing. Example: replicating forward contracts with futures. Arbitrage in multi-step and single-step worlds.

Cox-Ross-Rubinstein and Black-Scholes

The limit of European (vanilla) option prices in Cox-Ross-Rubinstein. Black-Scholes

formulae. CRR callibration revisited.

The Cox-Ross-Rubinstein model
European options: pricing and hedging in the binomial model. The underlying asset volatility. Callibrating the CRR model.

Multi-step models and the fundamental theorems
The separation lemma. The two fundamental theorems of financial mathematics in general multi-asset multi-step models.

Arbitrage pricing
The arbitrage-free markets assumption. The pricing operator in arbitrage-free markets. Arbitrage and the basic properties of option prices (monotonicity, convexity, parity, arbitrage bounds, Lipschitz property, symmetry). Arbitrage and replication. Parity and synthetic options.

Single step binomial model
Risk-neutral probabilities and arbitrage. The existence of a unique risk-neutral measure. Replicating and pricing derivatives. Fundamental theorems of Financial Mathematics in the simplest, single step, single asset setup.

Multi-asset single step models
General single step models. The separation lemma and the fundamental theorems of financial mathematics. Derivatives pricing in complete markets.

Conditional expectations in the discrete world
Conditional probabilities. Discrete stochastic processes. Describing time and information: partitions, sigma-fields and filtered probability spaces. Conditional expectations: definition and basic properties. Discrete martingales.

Auditorium classes

Na ćwiczeniach realizowane będą zadania ilustrujące tematykę wykładów.
Wykłady z przedmiotu są prowadzone w jezyku angielskim, natomiast ćwiczenia w języku polskim.

Teaching methods and techniques:

Lectures: Treści prezentowane na wykładzie są przekazywane w formie prezentacji multimedialnej w połączeniu z klasycznym wykładem tablicowym wzbogaconymi o pokazy odnoszące się do prezentowanych zagadnień.
Auditorium classes: Podczas zajęć audytoryjnych studenci na tablicy rozwiązują zadane wcześniej problemy. Prowadzący na bieżąco dokonuje stosowanych wyjaśnień i moderuje dyskusję z grupą nad danym problemem.

Warunki i sposób zaliczenia poszczególnych form zajęć, w tym zasady zaliczeń poprawkowych, a także warunki dopuszczenia do egzaminu:

Ćwiczenia z przedmiotu są zaliczane na podstawie kolokwiów i aktywności na zajęciach. Dokładne kryteria w tym względzie ustala prowadzący ćwiczenia. W wypadku nie uzyskania zaliczenia z ćwiczeń w pierwszym terminie studentom przysługuje jeden termin (jedno kolokwium) poprawkowe.

Zasady udziału w poszczególnych zajęciach, ze wskazaniem, czy obecność studenta na zajęciach jest obowiązkowa:

Lectures:
– Attendance is mandatory: Yes
– Participation rules in classes: Studenci uczestniczą w zajęciach poznając kolejne treści nauczania

zgodnie z syllabusem przedmiotu. Studenci winni na bieżąco zadawać pytania i wyjaśniać wątpliwości. Rejestracja audiowizualna wykładu wymaga zgody prowadzącego.
Auditorium classes:
– Attendance is mandatory: Yes
– Participation rules in classes: Studenci przystępując do ćwiczeń są zobowiązani do przygotowania się w zakresie wskazanym każdorazowo przez prowadzącego (np. w formie zestawów zadań). Ocena pracy studenta może bazować na wypowiedziach ustnych lub pisemnych w formie kolokwium, co zgodnie z regulaminem studiów AGH przekłada się na ocenę końcową z tej formy zajęć.

Method of calculating the final grade

1. Warunkiem koniecznym dopuszczenia do egzaminu jest posiadanie oceny pozytywnej z ćwiczeń.

2. Ocenę końcową OK wyznacza się na podstawie średniej ważonej SW obliczonej według wzoru
SW = 1/3 OC + 2/3 OE,
gdzie OC jest oceną uzyskaną z ćwiczeń,
a OE jest oceną uzyskaną z egzaminu.

3. Ocena końcowa OK. jest obliczana według algorytmu:
Jeżeli SW ≥ 4.75, to OK = 5.0 (bdb),
jeżeli 4.75 > SW ≥ 4.25, to OK = 4.5 (db),
jeżeli 4.25 > SW ≥ 3.75, to OK = 4.0 (db),
jeżeli 3.75 > SW ≥ 3.25, to OK = 3.5 (dst),
jeżeli 3.25 > SW ≥ 3.00, to OK = 3.0 (dst).

Sposób i tryb wyrównywania zaległości powstałych wskutek nieobecności studenta na zajęciach:

Student powinien zgłosić się do prowadzącego w celu ustalenia indywidualnego sposobu nadrobienia zaległości.

Prerequisites and additional requirements

Prerequisites and additional requirements not specified

Recommended literature and teaching resources

1) M.Capinski, T.Zastawniak, Discrete Models of Financial Markets, Cambridge University Press (2011)

2) M.Capinski, T.Zastawniak, Mathematics for Finance, Springer, London 2010.

3) S.Pliska, Introduction to Mathematical Finance. Discrete time models, Blackwell, Oxford 1997

4) R.Elliott, P.E.Kopp, Mathematics of Financial Markets, Springer 2006

5) S.Shreve, Stochastic Calculus for Finance I, The Binomial Asset Pricing Model, Springer 2004.2.

Scientific publications of module course instructors related to the topic of the module

1. M.Capinski, T.Zastawniak, Mathematics for Finance, Springer, London 2010.

2. M.Capinski, T.Zastawniak, Discrete Models of Financial Markets, Cambridge University Press (2011)

3. M.J. Capiński, Hedging conditional value at risk with options : short communication , European Journal of Operational Research (2015) vol. 242 iss. 2, s. 688–691.

4. M.J. Capiński, P.E. Kopp, Portfolio Theory and Risk Management, Cambridge University Press (2014)

5. M.J. Capińsk, T. Zastawniak, Numerical methods in finance with C++, Mastering Mathematical Finance. Cambridge: Cambridge University Press (2012).

Additional information

None