AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Module name: Operator Theory

Academic year: 2019/2020 Code: AMAT-2-105-MU-s ECTS credits: 6

Faculty of: Applied Mathematics

Field of study: Mathematics Specialty: Insurance Mathematics

Study level: Second-cycle studies Form and type of study: Full-time studies

Lecture language: English Profile of education: Academic (A) Semester: 1

Course homepage: http://home.agh.edu.pl/~rudol/OperatorTh/

Responsible teacher: dr hab. Rudol Krzysztof (rudol@agh.edu.pl)

Module summary

Student knows basic results from sectral theory of bounded (as well as unbounded) linear operators on Hilbert spaces and examples of their applicationsapplications.

Description of learning outcomes for module

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Connections with FLO	Method of learning outcomes verification (form of completion)
Social competence: is able to			
M_K001	-Student can estimate difficulties resulting in different approaches and knows basic sources of specialized results. He knows about the history of operator theory and some of its challanging open problems	MAT2A_K01, MAT2A_U22, MAT2A_K04, MAT2A_K06	Activity during classes, Oral answer
Skills: he can			
M_U001	-Student can use different methods of estimatimating action of a given operator T, depending on the chosen model. He /she can apply informations on the spectrum of T and its parts. Student can recognize structures related to systems of commuting operators, by using Banach and Hilbert space techniques	MAT2A_U22, MAT2A_U09, MAT2A_U13, MAT2A_U10	Activity during classes, Examination, Test, Oral answer
Knowledge: he knows and understands			

M_W001	-Student understands general pronciples governing the behaviour of linear rquations. He (she) knows basic methods of investigating linear operators in various situations and knows their relations to other branches of mathematics	MAT2A_U22, MAT2A_W04, MAT2A_W03	Examination, Test, Oral answer
M_W002	-Student knows basic results from sectral theory of bounded (as well as unbounded) linear operators on Hilbert spaces and examples of their applicationsapplications	MAT2A_W05, MAT2A_W07, MAT2A_W06, MAT2A_W04	Activity during classes, Examination, Test, Oral answer

Number of hours for each form of classes

	Form of classes										
Suma	Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
45	30	15	0	0	0	0	0	0	0	0	0

FLO matrix in relation to forms of classes

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Form of classes										
		Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
Social competence: is able to												
M_K001	-Student can estimate difficulties resulting in different approaches and knows basic sources of specialized results. He knows about the history of operator theory and some of its challanging open problems	+	-	-	-	-	-	-	-	-	-	-
Skills: he can												
M_U001	-Student can use different methods of estimatimating action of a given operator T, depending on the chosen model. He /she can apply informations on the spectrum of T and its parts. Student can recognize structures related to systems of commuting operators, by using Banach and Hilbert space techniques	+	+	-	-	-	-	-	-	-	-	-
Knowledge: he knows and understands												

M_W001	-Student understands general pronciples governing the behaviour of linear rquations. He (she) knows basic methods of investigating linear operators in various situations and knows their relations to other branches of mathematics	+	+	-	-	-	-	-	-	-	-	-
M_W002	-Student knows basic results from sectral theory of bounded (as well as unbounded) linear operators on Hilbert spaces and examples of their applicationsapplications	+	+	-	-	-	-	-	-	-	-	-

Student workload (ECTS credits balance)

Student activity form	Student workload
Udział w zajęciach dydaktycznych/praktyka	45 h
Preparation for classes	57 h
Realization of independently performed tasks	46 h
Examination or Final test	2 h
Summary student workload	150 h
Module ECTS credits	6 ECTS

Additional information

Module content

Lectures

Operator theory

1. Brief review of basic fact and terminology related to complete normed spaces and linear functionals. 2. Finite, infinite and block matrices as linear operators. Schur test for boundedness. The adjoint and the hermitian adjoint of an operator.
3. Finite rank and compact linear operators. Examples of integral operators. Operators with compact resolvent. Application to some differential equations.
4. Hilbert-Schmidt operators. Examples of integral operators.
5. Spectral representation of compact selfadjoint operators in Hilbert spaces.
6. Applications of spectral Thorem for compact operators. Polar decomposition.
7. Fredholm alternative, application to integral equations.
8. Sesquilinear forms corresponding to Hilbert space operators. Lax-Milgram theorem.
9. Numerical range. Comparing the numerical and spectral radii of an operator

10. General Spectral Thorem for normal operators.
11. Examples of spectral measures and applications of the Spectral Theorem.
12. Elementary properties of closed unbounded operators. Examples of differential operators. Cayley transform of symmetric oper 13.

Spectral theorem for unbounded operators.
14. Spectra of functions of operators.
15 Some recent results and open problems in operator theory.

Auditorium classes

tutorials on operator theory
Tutorials will discuss concrete examples and applications. The aim is to deepen understanding of the developed theory and to encourage students to try “hands on” approach to the problems.

Teaching methods and techniques:

Lectures: Treści prezentowane na wykładzie są przekazywane w formie prezentacji multimedialnej w połączeniu z klasycznym wykładem tablicowym wzbogaconymi o pokazy odnoszące się do prezentowanych zagadnień.
Auditorium classes: Podczas zajęć audytoryjnych studenci na tablicy rozwiązują zadane wcześniej problemy. Prowadzący na bieżąco dokonuje stosowanych wyjaśnień i moderuje dyskusję z grupą nad danym problemem.

Warunki i sposób zaliczenia poszczególnych form zajęć, w tym zasady zaliczeń poprawkowych, a także warunki dopuszczenia do egzaminu:

Dwa terminy zaliczeń poprawkowych.

Zasady udziału w poszczególnych zajęciach, ze wskazaniem, czy obecność studenta na zajęciach jest obowiązkowa:

Lectures:
– Attendance is mandatory: Yes
– Participation rules in classes: Studenci uczestniczą w zajęciach poznając kolejne treści nauczania zgodnie z syllabusem przedmiotu. Studenci winni na bieżąco zadawać pytania i wyjaśniać wątpliwości. Rejestracja audiowizualna wykładu wymaga zgody prowadzącego.
Auditorium classes:
– Attendance is mandatory: Yes
– Participation rules in classes: Studenci przystępując do ćwiczeń są zobowiązani do przygotowania się w zakresie wskazanym każdorazowo przez prowadzącego (np. w formie zestawów zadań). Ocena pracy studenta może bazować na wypowiedziach ustnych lub pisemnych w formie kolokwium, co zgodnie z regulaminem studiów AGH przekłada się na ocenę końcową z tej formy zajęć.

Method of calculating the final grade

30% of the final grade comes from assesement at tutorials, 70% from the exam.

Sposób i tryb wyrównywania zaległości powstałych wskutek nieobecności studenta na zajęciach:

Student powinien zgłosić się do prowadzącego w celu ustalenia indywidualnego sposobu nadrobienia zaległości.

Prerequisites and additional requirements

basic calculus courses including Lebesgue integral theory.
basic course in linear algebra

Recommended literature and teaching resources

1. N.I.Ahiezer. I.M. Glazman, Theory of Lienar Operators In Hilbert Spaces, Ungar, N.Y., 1961
2. J. B. Conway, Course in functional analysis, Springer-Verlag, New York, 1985.
3. G. K. Lax, Functional Analysis, Warszawa, Wiley-Interscience, 2002.

4. G. Pedersen, Analysis Now, Springer-Verlag, N.Y. 1989.
5. W. Rudin, Functional analysis, McGraw-Hill, 1973.
Some problems, hints and examples will be provided on my webpage’s section devoted for this course (section under construction as on 21. 05. 2013)

Scientific publications of module course instructors related to the topic of the module

1) K. Rudol, Extensions of the Foias – Mlak spectral mapping theorem, Univ. Iagell. Acta Math. (Zeszyty Naukowe UJ) 34, (1997), 101-111.

2) K. Rudol, Some results related to Beurling’s theorem. Univ. Iagell. Acta Math. (Zeszyty Naukowe UJ) 38 (2000) Fasc. 38, 290-298.

3) K. Rudol, Corona theorem and isometries Opuscula Math.24 (2004).

4) Spectra of subnormal pairs, Opuscula Math. 27, (2007), 301-304.

5) Z.Ambrozinski, K. Rudol, Matrices defined by frames, Opuscula Math. 29, (2009), 365-375.

6) K. Rudol, Matrices related to some Fock space operators, Opuscula Math. 31, (2011), 289-296

7) M.Kosiek, K. Rudol, Dual algebras and A-measures, Journal of function spaces.

Additional information

None