



Module name: Fractional calculus and its applications in technical sciences

Academic year: 2019/2020 Code: ZSDA-3-0096-s ECTS credits: 3

Faculty of: Szkoła Doktorska AGH

Field of study: Szkoła Doktorska AGH Specialty: —

Study level: Third-cycle studies Form and type of study: Full-time studies

Lecture language: English Profile of education: Academic (A) Semester: 0

Course homepage: —

Responsible teacher: prof. dr hab. inż. Leszczyński Jacek (jale@agh.edu.pl)

Module summary

The course utilises fractional derivatives and integrals in order to illustrate mathematically complex processes and phenomena in nature. PhDs learn fundamentals in self-study programme that use the fractional operators in their scientific activities. In the first part we are able to make some introduction. Next part covers simple calculus and manipulations. The last part involves self-proposed formulae or mathematical models which could be useful in PhD developments.

Description of learning outcomes for module

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Connections with FLO	Method of learning outcomes verification (form of completion)
Social competence: is able to			
M_K001	PhD can work in a group in order to elaborate mathematical illustration of complex phenomena and processes	SDA3A_K01	Oral answer
Skills: he can			
M_U001	PhD can calculate fractional differentiation and fractional integration	SDA3A_U01, SDA3A_U02	Case study
Knowledge: he knows and understands			
M_W001	PhD knows fundamentals within fractional derivatives and integrals	SDA3A_W02, SDA3A_W01	Activity during classes
M_W002	PhD knows ordinary and partial fractional differential equations and methods of their creations	SDA3A_W02	Activity during classes

Number of hours for each form of classes

Suma	Form of classes										
	Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
30	0	0	0	0	30	0	0	0	0	0	0

FLO matrix in relation to forms of classes

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Form of classes										
		Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
Social competence: is able to												
M_K001	PhD can work in a group in order to elaborate mathematical illustration of complex phenomena and processes	-	-	-	-	+	-	-	-	-	-	-
Skills: he can												
M_U001	PhD can calculate fractional differentiation and fractional integration	-	-	-	-	+	-	-	-	-	-	-
Knowledge: he knows and understands												
M_W001	PhD knows fundamentals within fractional derivatives and integrals	-	-	-	-	+	-	-	-	-	-	-
M_W002	PhD knows ordinary and partial fractional differential equations and methods of their creations	-	-	-	-	+	-	-	-	-	-	-

Student workload (ECTS credits balance)

Student activity form	Student workload
Udział w zajęciach dydaktycznych/praktyka	30 h
Preparation for classes	5 h
przygotowanie projektu, prezentacji, pracy pisemnej, sprawozdania	20 h
Inne	5 h
Summary student workload	60 h
Module ECTS credits	3 ECTS

Additional information

Module content

Conversation seminar

The following topics are composed with two-working-hours lectures:

1. Fundamentals, nomenclature, special functions and their properties.
2. Fractional operators: definitions, properties, basic calculations
3. Numerical representation of fractional integrals and derivatives, discrete schemes
4. Integral transforms and ordinary-fractional differential equations
5. Fractional Taylor series, examples, own trainings
6. Compositions of fractional operators, mixture of left- and right-sided forms, spatial and temporal and spatio-temporal operators
7. Fractional ordinary and "partial" differential equations, initial and boundary conditions, numerical representations
8. Applications in classical mechanics, own trainings and propositions
9. Analysis and applications of anomalous diffusion, own trainings and propositions
10. Applications in electrical engineering – own trainings and propositions
11. Applications in mathematical and physical sciences, mathematical modelling/illustrations of complex processes and phenomena in nature – own trainings and propositions
12. Applications in material sciences – own trainings and propositions
13. Applications in energy sciences – own trainings and propositions
14. Applications in bioengineering sciences – own trainings and propositions
15. Summarizing results, contemporary applications, advantages and disadvantages of fractional operators, unsolved problems

Teaching methods and techniques:

Conversation seminar: The introductory topics are presented by the lecturer in the form of multimedia and whiteboard lectures. Practical topics are trained jointly with PhDs. Additionally, PhD prepares self-motivated results for discussion and criticism.

Warunki i sposób zaliczenia poszczególnych form zajęć, w tym zasady zaliczeń poprawkowych, a także warunki dopuszczenia do egzaminu:

Mandatory attendance on the course, self-activity on lectures, self-prepared study.

Zasady udziału w poszczególnych zajęciach, ze wskazaniem, czy obecność studenta na zajęciach jest obowiązkowa:

Conversation seminar:

- Attendance is mandatory: Yes
- Participation rules in classes: PhDs by entering the lectures are required to prepare themselves in the scope indicated each time by the lecturer. Assessment of PhD's work is based on oral discussion and written report in the final part of the course.

Method of calculating the final grade

Final grade is associated within PhD activity in conversation seminar (20%) and takes into account self-prepared study in the frame of the first utilisation of fractional calculus (80%).

Sposób i tryb wyrównywania zaległości powstałych wskutek nieobecności studenta na zajęciach:

PhD must elaborate additional self-study in the form of report.

Prerequisites and additional requirements

Algebra, mathematical analysis, ordinary and partial differential equations, field theory, physics.

Recommended literature and teaching resources

1. Abramowitz M., Stegun I.A., Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Dover Publ., New York 1964
2. Bagley R.L., Torvik P.J., On the appearance of the fractional derivative in the behavior of real materials, J. Appl. Mech. 51(4), 1984, 294-298
3. Kaczorek T., Reachability of cone fractional continuous-time linear systems, Int. J. Appl. Math. Comput. Sci. 19(1), 2009, 89-93
4. Kilbas A., Srivastava H., Trujillo J., Theory and Applications of Fractional Differential Equations, North-Holland Mathematics Studies 204, Elsevier, 2006
5. Klafter J., Sokolov I.M., First steps in random walks, Oxford Univ. Press, Oxford, UK, 2017
6. Klimek M., On Solutions of Linear Fractional Differential Equations of a Variational Type, Monograph no. 172, Publ. Office of Czestochowa University of Technology, Czestochowa 2009
7. Leszczynski J.S., An introduction to fractional mechanics, Monograph no. 198, Publ. Office of Czestochowa University of Technology, Czestochowa 2011, 128 pages
8. Magin R.L., Fractional Calculus in Bioengineering, Begell House Inc., Redding 2006
9. Miller K.S., Ross B., An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley and Sons, New York 1993
10. Oldham K.B., Spanier J., The Fractional Calculus, Academic Press, New York 1974
11. Osler T.J., Taylor's series generalized for fractional derivatives and applications, SIAM J. Math. Anal. 2, 1971, 37-48
12. Ostalczyk P., An Outline of Differential-Integral Calculus of Fractional Order. Theory and Applications in Automation, Wyd. Pol. Łódzkiej, Łódź, 2008 (in Polish)
13. Podlubny I., Fractional Differential Equations, Academic Press, San Diego 1999
14. Povstenko Y.Z., Fractional heat conduction equation and associated thermal stress, J. Thermal Stresses 28(1), 2005, 83-102
15. Sabatier J., Agrawal O.P., Machado J.A.T., Advances in Fractional Calculus, Springer, The Netherlands 2007
16. Samko S.G., Kilbas A.A., Marichev O.I., Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach Science Publishers, Switzerland 1993
17. Wheatcraft S.W., Meerschaert M.M., Fractional conservation of mass, Advances in Water Resources 31, 2008, 1377-1381

Scientific publications of module course instructors related to the topic of the module

1. Ciesielski M., Leszczynski J., Numerical solutions of a boundary value problem for the anomalous diffusion equation with the Riesz fractional derivative, Proc. of the 16th International Conference on Computer Methods in Mechanics CMM-2005, Czestochowa 2005

2. Ciesielski M., Leszczynski J., Numerical treatment of an initial boundary value problem for fractional partial differential equations, *Signal Processing* 86(10), 2006, 2619-2631
3. Ciesielski M., Leszczynski J., Numerical solutions to boundary value problem for anomalous diffusion equation with Riesz-Feller fractional operator, *J. Theoret. and Appl. Mech.* 44(2), 2006, 393-403
4. Leszczynski J.S., An introduction to fractional mechanics, Monograph no. 198, Publ. Office of Czestochowa University of Technology, Czestochowa 2011, 128 pages
5. Leszczynski J.S., Ciesielski M., A numerical method for solution of ordinary differential equations of fractional order, *Lect. Not. in Comp. Sci.* 2328, 2002, 695-702
6. Leszczynski J.S., A discrete model of a two-particle contact applied to cohesive granular materials, *Gran. Matt.* 5(2), 2003, 596-605
7. Leszczynski J.S., Using the fractional interaction law to model the impact dynamics of multiparticle collisions in arbitrary form, *Phys. Rev. E* 70, 2004, 051315
8. Leszczynski J.S., Blaszczyk T., Modeling the transition between stable and unstable operation while emptying a silo, *Granular Matter*, 2011, DOI 10.1007/s10035-010-0240-5

Additional information

Lecturer offers additional help for PhDs taking into account utilisation of fractional calculus in their scientific activity that make new results for publication.