

**AGH**AGH UNIVERSITY OF SCIENCE  
AND TECHNOLOGY

Module name: Ergodic theory

Academic year: 2019/2020 Code: ZSDA-3-0254-s ECTS credits: 4

Faculty of: Szkoła Doktorska AGH

Field of study: Szkoła Doktorska AGH Specialty: —

Study level: Third-cycle studies Form and type of study: Full-time studies

Lecture language: English Profile of education: Academic (A) Semester: 0

Course homepage: —

Responsible teacher: prof. dr hab. Oprocha Piotr (oprocha@agh.edu.pl)

### Module summary

The lecture is introduction to ergodic theory. The aim of classes is to increase understanding of basic properties and definitions presented during lecture, by solving short problems.

### Description of learning outcomes for module

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Connections with FLO	Method of learning outcomes verification (form of completion)
Social competence: is able to			
M_K001	Student is able to apply advanced methods of ergodic theory to solve specific problems.	SDA3A_U01	Activity during classes
Skills: he can			
M_U001	Student has sufficient knowledge to understand research results at the edge of ergodic theory and dynamical systems.	SDA3A_U02	
Knowledge: he knows and understands			
M_W001	Student knows main measure-theoretic tools used in modern theory of dynamical systems.	SDA3A_W02	
M_W002	Student is aware of main research directions in modern ergodic theory, and has sufficient knowledge to perform his own research in this field.	SDA3A_W01	

## Number of hours for each form of classes

Suma	Form of classes										
	Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
60	30	30	0	0	0	0	0	0	0	0	0

## FLO matrix in relation to forms of classes

MLO code	Student after module completion has the knowledge/ knows how to/is able to	Form of classes										
		Lectures	Auditorium classes	Laboratory classes	Project classes	Conversation seminar	Seminar classes	Practical classes	Fieldwork classes	Workshops	Prace kontrolne i przejściowe	Lektorat
Social competence: is able to												
M_K001	Student is able to apply advanced methods of ergodic theory to solve specific problems.	+	+	-	-	-	-	-	-	-	-	-
Skills: he can												
M_U001	Student has sufficient knowledge to understand research results at the edge of ergodic theory and dynamical systems.	+	+	-	-	-	-	-	-	-	-	-
Knowledge: he knows and understands												
M_W001	Student knows main measure-theoretic tools used in modern theory of dynamical systems.	+	+	-	-	-	-	-	-	-	-	-
M_W002	Student is aware of main research directions in modern ergodic theory, and has sufficient knowledge to perform his own research in this field.	+	+	-	-	-	-	-	-	-	-	-

## Student workload (ECTS credits balance)

Student activity form	Student workload
Udział w zajęciach dydaktycznych/praktyka	60 h
Preparation for classes	30 h
Realization of independently performed tasks	10 h
Examination or Final test	2 h
Contact hours	5 h
Summary student workload	107 h
Module ECTS credits	4 ECTS

## Additional information

### Module content

#### Lectures

1. Measure preserving transformations. Poincare recurrence theorem.
2. Invariant and ergodic measures. Notions of mixing.
3. Koopman operator and von Neumann ergodic theorem.
4. Birkhoff ergodic theorem.
5. Isomorphisms and spectral properties.
6. Invariant measures for continuous maps.
7. Entropy (metric and topological). Variational principle.

#### Auditorium classes

Students will apply new techniques, theorems and constructions learned during lectures to solve a given problems.

#### Teaching methods and techniques:

Lectures: Classical lecture on blackboard.

Auditorium classes: Presentation and discussion on correctness of solutions. Comparison of different solving techniques used by various students.

#### Warunki i sposób zaliczenia poszczególnych form zajęć, w tym zasady zaliczeń poprawkowych, a także warunki dopuszczenia do egzaminu:

Students solve exercises given on lecture and present own solutions during classes. On that basis students are evaluated during classes. Positive mark from classes entitles to oral exam.

#### Zasady udziału w poszczególnych zajęciach, ze wskazaniem, czy obecność studenta na zajęciach jest obowiązkowa:

Lectures:

- Attendance is mandatory: Yes
- Participation rules in classes: Nie określono

Auditorium classes:

- Attendance is mandatory: Yes
- Participation rules in classes: Students present own solutions to problems provided during lecture.

### **Method of calculating the final grade**

Final grade is a weighted average of exam (0.5) classes (0.5)

### **Sposób i tryb wyrównywania zaległości powstałych wskutek nieobecności studenta na zajęciach:**

The student should report to the lecturer to determine the individual way of catching up.

### **Prerequisites and additional requirements**

Basic knowledge of probability theory (probability measures) and mathematical analysis.

### **Recommended literature and teaching resources**

P. Walters, An introduction to ergodic theory, Springer-Verlag, New York-Berlin, 1982.

K. Petersen, Ergodic theory, Cambridge University Press, Cambridge, 1989.

W. Parry, Entropy and generators in ergodic theory, W. A. Benjamin, Inc., New York-Amsterdam, 1969.

M. Einsiedler, T. Ward, Ergodic Theory with a view towards Number Theory, Springer-Verlag 2010.

### **Scientific publications of module course instructors related to the topic of the module**

P. Oprocha, P. Potorski, P. Raith, "Mixing properties in expanding Lorenz maps", Adv. Math., 343 (2019), 712-755.

J. Boroński, J. Kupka and P. Oprocha, "Mixing completely scrambled system exists", Erg. Th. Dynam. Syst., 39 (2019), 62-73.

P. Oprocha, "Double minimality, entropy and disjointness with all minimal systems", Discrete Contin. Dyn. Syst., 39 (2019), 263–275.

C. Good, P. Oprocha, M. Puljiz, "Shadowing, asymptotic shadowing and s-limit shadowing", Fund. Math., 244 (2019), 287-312.

J. Boroński, A. Clark and P. Oprocha, "A compact minimal space  $Y$  such that its square  $Y \times Y$  is not minimal", Adv. Math., 335 (2018), 261-275.

W. Brian, P. Oprocha, "Ultrafilters and Ramsey-type shadowing phenomena in topological dynamics", Israel J. Math., 227 (2018), 423-453.

Y. Dong, P. Oprocha and X. Tian, "On the irregular points for systems with the shadowing property", Erg. Th. Dynam. Syst., 28 (2018), 2108-2131.

### **Additional information**

None